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Dynamical Modeling for Studying Self-Regulatory Processes

An Example From the Study of Religious Development Over the Life Span

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The development of religious faith and practice over the life course has been a topic of interest to psychological theorists for over a century (Fowler, 1981; Hall, 1904). In recent years, social scientists have applied quantitative methods from modern developmental science to questions regarding the development of religious feeling, belief, and behavior over the adult life course (Argue, Johnson, & White, 1999; McCullough, Enders, Brion, & Jain, 2005; Sasaki & Suzuki, 1987). Many of the models that have been proposed by theorists such as Hall and Fowler, and the empirical tests of developmental hypotheses, have relied upon the assumption that a person's religiousness may change (or not) as a function of how old he or she is. In other words, these models and tests have assumed that it is something about getting older per se that controls religious development.

Other scientists have given a role to the effects of life events on religious development (Bahr, 1970; Ingersoll-Dayton, Krause, & Morgan, 2002; Kelley-Moore & Ferraro, 2001; Sherkat, 1998; Stolzenberg, Blair-Loy, & Waite, 1995), providing an understanding of how the vicissitudes of life can influence people's religious beliefs and behaviors. Such research has explored, for instance, how normative events in the life cycle—going to college, getting married, having children, getting divorced, the empty nest, the death of a spouse, and the development of physical disability—can produce growth and decline in religiousness.

Together, the age-dependent perspective and the life-events perspective on religious development have shed important light on the development of religiousness over the life course. However, another developmental perspective has been ignored: A perspective based on the notion that individuals actively govern their religious belief and behavior according to an internal guidance system. This internal guidance system might be thought of as a coordinated set of psychophysiological processes comprising reference

values, perceptions of one's social world, programs for behavioral action, and feedback loops that seeks to produce in individuals a degree of religiousness that enables them to flourish in their social environments, modify environments that are not working for them, and feel comfortable with what they believe and how they interact with the world.

This third perspective is not mutually exclusive of the age-dependent and life-events models of religious development, but it is distinct from them. This chapter is the first of which we are aware to extend theory and research on religious development by proposing and testing the idea that religiousness does not simply change as a function of age, or as a function of life circumstances, but rather as the result of self-regulation.

The proposition that humans are selfregulating creatures that use information about how they are changing to exert change upon their own physiological, psychological, and behavioral states must surely be one of the most foundational assumptions of any positive psychology of human behavior. However, this assumption is hardly unique to positive psychology. In one respect, the assumption that human beings are self-regulatory is utterly uncontroversial, for every organism must regulate its behavior to ensure survival. At the levels of cells, tissue, and organs, many human physiological functions are governed by homeostatic mechanisms. Blood pressure, heart rate, hunger, water balance, temperature, and respiration are all selfregulated as well, as are the body's responses to psychological and physical stress (Selve, 1956).

Many contemporary theorists in personality, social, developmental, and clinical psychology assume that psychological and behavioral processes are governed by self-regulatory processes as well. In their book On the Self-Regulation of Behavior, Carver and Scheier (1998) proposed "that human behavior is a continual process of moving toward, and away from, various kinds of mental goal representations, and that this movement occurs by a process of feedback control. This view treats behavior as the consequence of an internal guidance system inherent in the way living beings are organized ... we refer to the guidance process as a system of selfregulation" (p. 2). Some of these self-regulatory processes can be conceptualized as preferred set points (e.g., weight, happiness), with individuals' biobehavioral systems attempting to keep the values of a system within a small zone of deviation from the set points. For example, it is

often assumed that the biological system that controls body mass and the affective system that controls mood both regulate people toward certain ideal weights or ideal balances of positive and negative affect. In addition, the pursuit of goals, which is often a conscious process involving volitional effort, can also be viewed through the lens of self-regulation—the process by which individuals actively work to change their behavior to make it conform to their standards (Carver & Scheier, 1998; Higgins, Grant, & Shah, 1999).

Although theorists and researchers in positive psychology have also recognized that selfregulation is an important aspect of many forms of strength, resilience, and adaptation, few researchers in psychology and the behavioral sciences, and perhaps none in the domain of positive psychology, have taken advantage of the considerable advances in multivariate statistics that allow for tests of self-regulatory processes (Boker & Nesselroade, 2002; McArdle & Hamagami, 2003; Oud & Jansen, 2000). Such tools have been used to shed light on the millisecondto-millisecond regulation of posture in infants (Boker, 2001), the day-to-day regulation of psychological well-being among recent widows (Bisconti, 2001), and the year-to-year regulation of tobacco and alcohol use among adolescents (Boker & Graham, 1998). The goal here is to introduce some of these tools to researchers interested in positive psychology and to illustrate their utility for testing hypotheses regarding the extent to which a given psychological process is governed by a self-regulatory mechanism. Many constructs that are of interest to the emerging positive psychology field are based on the notion that human behavior either is guided by selfregulatory processes (Masten & Reed, 2002) or can be conceptualized themselves as self-regulatory processes (McCullough & Witvliet, 2002), so understanding how multivariate methods can be used to study those self-regulatory processes could be a boon to scientific progress in positive psychology.

In the pages that follow, we (a) introduce some basic concepts regarding self-regulation and the differential equations that can be used to test self-regulation processes with longitudinal data; (b) introduce some theoretical background that sets the stage for considering how religious or spiritual development may be governed by self-regulation; (c) report the results of a study in which we explored self-regulatory dynamics in religious development; (d) discuss what these

analyses might suggest about religious development and future work on this particular topic; and (e) offer some general recommendations to researchers who want to use these techniques in their own work.

Modeling Longitudinal Data From a Self-Regulatory Perspective

Suppose we are interested in a psychological construct that is hypothesized to regulate itself. In other words, suppose that the construct may undergo changes such that future values of the construct's indicator variables are dependent on the current value of those same variables. For instance, when the construct was far from a preferred equilibrium value or set point, change might occur such that the construct would tend to return toward the equilibrium value. Or perhaps there might be a preference for slow change rather than rapid change. Then changes that occurred too rapidly might be reduced or damped. In this case, the change itself would be changed by the self-regulation.

It is self-evident that in the preceding paragraph we relied heavily on the word change. Additionally, one should note that when speaking of self-regulatory processes, one is naturally led to propose models in which quantifiable measures of change are the outcome or predictor variables. For that reason, it is important to be specific about what sort of change we mean and exactly how to quantify it. To observe change, some interval of time must elapse. Thus, observed change is always relative to some interval of time. But, if we make the assumption that the construct of interest changes continuously over time, we can calculate a convenient abstract concept: instantaneous change, a derivative of the construct with respect to time.

Consider the trajectory of continuous change of a construct *X* over some interval of time from t = 0 to t = 10 plotted as a gray curve in figure 26.1. From t=0 to t=2, the construct is increasing and becoming farther from its equilibrium, followed by a period of decline in the interval from t = 2 to t = 7.5, and then finally a period of increase again. There are several observations that can be made about this trajectory. It is apparent that the construct does not stray far from its equilibrium since the slope of the curve is changing from positive to negative to positive again in a regular way. That is, when the trajectory strays too far from equilibrium, the slope changes sign and the trajectory heads back toward equilibrium.

The arrows in figure 26.1 plot the slope of the trajectory at each of seven points (*a*, *b*, *c*, ..., *g*). Another way to think about these arrows is that they plot the predicted change in the construct X at a time t if we thought there was only linear change in X. The more these linear predictions diverge from the trajectory, the more curvature in the trajectory. For instance, at points b and fthere is a great deal of mismatch between the trajectory and the linear prediction. But at point d the linear prediction is quite good. For this reason, in dynamical systems we do not speak of figure 26.1 as being a nonlinear system even though the trajectory is different than a line. Recall that sometimes the trajectory curves and sometimes it does not—sometimes a line is a good approximation and sometimes it is not. In dynamical systems we ask, "Can the change, both slope and curvature, be accounted for by a system of linear equations?" If so, we call it a linear system, and if not we call it nonlinear.

Note that in figure 26.1, the farther a point is from equilibrium, the greater the mismatch between the arrow and the trajectory. In fact, in this example there is a linear relationship between the

Figure 26.1 A construct *X* fluctuates around its equilibrium (0 on the ordinate axis) for the time interval t = 0 to t = 10. Arrows indicate slope at each point, that is, the first derivative of the curve at that point. Change in the slope, that is, the second derivative of the curve, is indicated by the deviation between the arrowhead and the trajectory.



deviation from equilibrium of a chosen point on the trajectory and the second derivative of the trajectory, that is, the curvature, at that chosen point.

Now consider the five trajectories plotted in figure 26.2. Each of these trajectories has the same linear relationship between its displacement and its second derivative. But each curve has a different starting value at time t = 0. In psychological terms, we would say that there were no individual differences in the parameters of the self-regulation, but there were individual differences in initial conditions, that is, the values of the variables at time t = 0. In this case, the differences were in the initial displacement from equilibrium, but there were no individual differences in slope at time t = 0.

Of course, in real data, trajectories will never be so smooth and similar to one another. We must find ways to statistically aggregate data in such a way that intraindividual change and interindividual similarities are not obscured by the aggregation. If we were to simply average all of the slopes in figure 26.2, we would find that there was a modest negative slope. In fact, one can simply draw a line from the mean value of the displacement at t=0 to the mean value of the displacement at t = 10 and this line would have exactly the mean of all of the slopes of all of the trajectories over the entire interval. Such a procedure tells us no more than would a pretest/ posttest design-in either case we would learn nothing about the highly patterned intraindividual variability in these data.

In these data, such a mean slope is extremely misleading. If we chose to end our experiment at t = 8, the mean slope would be more negative. But if we chose to end our experiment at t = 6, the mean slope would be very close to zero. Finally, suppose we ended our experiment at t = 3. Now the line connecting the mean displacement

at t=0 and the mean displacement at t=3 is strongly positive. Which mean slope are we to believe? Since each individual's curve changes sign from positive to negative to positive again, the aggregation method must not aggregate over too much time or this patterned intraindividual variability will be obscured.

Suppose instead we were to aggregate a mean slope within a small interval of time and within a small set of values of displacement. For instance, in figure 26.3 a grid is superimposed and the mean slope is taken within each box of the grid for all trajectories crossing that box in the grid. The plot of the dark line segments in figure 26.3 is called an empirical slope field (Boker & McArdle, 2005). In this plot, the changes in sign of the slope become apparent. Recall that there were no individual differences in slope in the initial conditions at time t = 0. Note that the line segments in a column are similar to one another. Similarity in rows or in columns are a clue that helps in guiding model building in dynamical systems. We wish to build a model in which we can test for reliable relationships between derivatives of a system. Aggregating and plotting derivatives (in this case the first derivative) against a variable or variables can help reveal which of these relationships might be strong.

To create an empirical slope field, one does not need to make an assumption about where the equilibrium might be. In other words, the zero on the ordinate axis in figure 26.3 could change and the slope field would not change. This means that we can sometimes use a slope field to help determine a likely value for the equilibrium.

A word of caution is in order here: Sometimes aggregating over displacement and time is not an effective method for determining a likely equilibrium value. Suppose there were individual differences in equilibrium value; that is to say, every individual might have a separate set point



Figure 26.2 Five trajectories with the same relationship between displacement and curvature, but with different starting values.

Figure 26.3 An empirical slope field. Average slope of all trajectories within each box is plotted as a single line segment centered in that box. When no trajectories cross a box, no line segment is plotted.

on the construct *X*. In this case, aggregating over individuals can obscure patterned intraindividual variability (i.e., the dynamics of selfregulation) that would be observed if the true individual equilibria were known. Random coefficients models provide one method for testing for individual differences in equilibria (Boker & Bisconti, 2006).

We will next consider a simple linear differential equations model that could account for self-regulating fluctuations about an equilibrium value—a second-order linear differential equation in which the curvature is a linear combination of the slope and displacement. This is often called a damped linear oscillator model because it approximates the motion of a pendulum with friction. Suppose we have measured a construct *X* at *N* occasions separated by an interval *s*. Then the damped linear oscillator model for the time series data $X = \{x_1, x_2, x_3, ..., x_N\}$ can be written

$$\ddot{\mathbf{x}}_t = \eta \mathbf{x}_t + \zeta \dot{\mathbf{x}}_t + e_t \tag{1}$$

where \ddot{x}_t and \dot{x}_t are the second and first derivatives of x with respect to time, η is a coefficient related to the frequency of the oscillation, ζ is a coefficient related to how quickly the oscillations



are damped to equilibrium, and e_t is a residual term.

Each of the 15 gray trajectories in figure 26.4 is the result of applying Equation 1 with the coefficients $\eta = -0.2$ and $\zeta = -0.3$ and the error term equal to zero: a completely deterministic system with no individual differences in coefficients. If there are no individual differences in coefficients, then why do the trajectories look so different from each other? These trajectories differ only in their initial conditions. There are five different values of initial displacement $\{-1, -0.4, 0.4, 0.8, 1.2\}$ and three different values of initial slope $\{0, 3, 5\}$.

One way to analyze these data would be to fit latent growth curves to the trajectories. In such a case, we would reject the hypothesis that the coefficients of these trajectories are equal. Latent growth curve analysis is widely used and is appropriate for many problems. However, growth curve analysis has drawbacks when used to specify models for self-regulation. First, as seen in this example, growth curves can confound individual differences in initial conditions with individual differences in coefficients. The way these 15 simulated individuals self-regulate is identical; only their initial conditions differ.

Figure 26.4 Fifteen trajectories generated from Equation 1. Every trajectory has the same equilibrium and the same coefficients $\eta = -0.2$ and $\zeta = -0.3$. However, there are individual differences both in the deviation from equilibrium at time t = 0 and in the slope at time t = 0.



Second, the parameters obtained from a growth curve analysis relate to either an aggregate trajectory or how each individual's trajectory conforms to the particular hypothesized model. These results say nothing about what might have happened if the individual had started at another initial condition.

In contrast, fitting a differential equation model to longitudinal data specifically distinguishes between model coefficients and initial conditions in such a way that a family of trajectories is implied. Each possible initial condition has associated with it a trajectory. In this way we can explore questions such as, "How similar would two individuals' trajectories be if they self-regulated differently (had different coefficients) but the same initial conditions?" Or the opposite question could be asked: "How similar would two individuals' trajectories be if they self-regulated in the same way, but had different initial conditions?"

Superimposed on figure 26.4 is an empirical slope field. Note that by simply focusing on the line segments of the slope field, we can estimate the equilibrium for this equation to be somewhere near zero. Note that except for the first interval, from t=0 to t=2, the slopes above zero are negative and the slopes below zero are positive. This method is used to provide a preliminary estimate of an equilibrium value for the example data analyzed later in this chapter.

Given data from trajectories such as those plotted in figure 26.4, we can fit Equation 1 if we can estimate the derivatives of the construct from the observed time series. In the example, we will use local linear approximation (Boker & Nesselroade, 2002) to estimate these derivatives and fit multilevel models in order to account for potential individual differences in self-regulation (Boker & Ghisletta, 2001).

Religious Development as a Self-Regulatory Process

At this point, we illustrate some of the concepts introduced above by analyzing some data drawn from the real world. One psychological domain in which we can ask questions about self-regulation that should be of interest to positive psychology is in the area of religious and spiritual development (Mattis, 2004; Pargament & Mahoney, 2002; Tsang & McCullough, 2003). Although researchers have found evidence that religious and spiritual changes over the life course arise from aging per se (Argue et al., 1999; Sasaki & Suzuki, 1987) and the influence of external life events (Bahr, 1970; Ingersoll-Dayton et al., 2002; Kelley-Moore & Ferraro, 2001; Sherkat, 1998; Stolzenberg et al., 1995), spiritual or religious change over the life course probably does not result solely from agerelated development and external forces acting upon individuals. To some extent, spiritual and religious changes may also be caused by selfregulation processes that are intrinsic to individual functioning. Just as heart rate has an intrinsic variability independently of any forces acting upon the organism, spirituality and religiousness may vary as a function of an intrinsic self-regulation process. As we mentioned previously, self-regulation is a process by which an organism uses information about the way its behavior is changing to modify future behavior (Boker, 2001). Insofar as the human tendency toward religiousness is ordered in such a way that the current state of an individual's religious system (i.e., the person's religiousness at time *t*) predicts the future behavior of the system (i.e., the same person's religiousness at time t+1), this system is said to possess intrinsic dynamics (Boker & Graham, 1998), and we can posit a selfregulatory mechanism that works to achieve stability or equilibrium (Boker, 2001).

Because religiousness is at least partially based upon genetic effects (for review see D'Onofrio, Eaves, Murrelle, Maes, & Spilka, 1999) and strong effects for socialization processes (e.g., Flor & Knapp, 2001) that may set people's preferences for certain optimal degrees of religiousness, it is reasonable to expect that adults' religiousness is subject to self-regulatory processes that effectively pull their levels of religiousness toward points of equilibrium. As a result, even though aging and the vicissitudes of life (e.g., bereavement, health problems, marriage, child rearing, etc.), might create fluctuation in people's spirituality or religiousness over time, an intrinsic self-regulatory process may also be active in directing the extent to which people define their lives in terms of religion and engage in religious activities.

Intrinsic dynamics are often modeled in terms of differential equations in which accelerations in a dependent variable (i.e., the extent to which changes in religiousness at any given point in time are speeding up or slowing down, operationalized as the second derivatives of the individual's observed scores) are regressed simultaneously upon the measured value of the

variable (e.g., actual scores of an individual's religiousness at a given point in time) and the rate of change in those measured values (i.e., the first derivatives of the individuals' observed scores). One dynamical model that may be of particular value for shedding light on self-regulatory processes underlying religious and spiritual change is a damped linear oscillator with a single point attractor (in which scores oscillate around one or more point attractors, with variation around the point attractors gradually becoming smaller and smaller, like a pendulum with friction).

A second dynamical model that has a substantive psychological interpretation is a damped linear oscillator model with two point attractors rather than one. In positing such a model, one proposes that religious development involves oscillation caused by two points of stability or equilibrium that exert simultaneous influences on people's religiousness over time.

In the context of religious and spiritual development, these two dynamical models have substantive psychological interpretations (for a fuller treatment, see Boker & Graham, 1998). A damped linear oscillator model with a single point attractor corresponds to a self-regulatory system in which an individual's level of religiousness oscillates between values that are higher than optimal and values that are lower than optimal, eventually converging closer and closer to a point of equilibrium. We might imagine an individual who begins adulthood with a higher-than-optimal level of religiousness (with optimal defined as the value that maximizes person-environment fit in the broadest sense), but finds the social consequences of this high level of religiousness to be unpleasant (i.e., it alienates friends and family members). As a result, a self-regulatory process pulls the individual's religiousness toward equilibrium but overshoots this optimum. At this lower-than-optimal level of religiousness, the individual experiences problems adapting to his or her social environment that are experienced as unpleasant and which being more religious is perceived to remedy. In response, the individual's self-regulatory process then attracts him or her to an increased level of religiousness, thereby overshooting the optimum but not to the same degree as before. This process of oscillation around an optimum, with better and better approximation of the optimum in each cycle, is posited to continue until a point of equilibrium is reached.

A second-order model with two attractors implies that individuals' religious trajectories

are produced by oscillation around two rather than one point attractors. This might occur in a system in which it is possible to develop a high degree of person-environment fit by settling upon either of two values of religiousness. In a highly religious society such as the United States, in which one can easily find social structures and social relations that reinforce both very high and very low levels of religiousness, it is perhaps more plausible to conceive of religiousness as a two-attractor system, with people eventually settling upon very high or very low levels of religiousness over time.

A damped linear oscillator model with a single point of attraction can be specified in terms of the causal effects of a religious variable's measured values and first derivative (i.e., rate of change) on its second derivative (i.e., rate of acceleration). Referring back to Equation 1, we can write:

$$\ddot{\mathbf{x}}_t = \eta \mathbf{x}_t + \zeta \dot{\mathbf{x}}_t + e_t \tag{1}$$

where \ddot{x}_t acceleration in the rate at which religiousness is changing at time t; $\zeta =$ the coefficient of damping (i.e., the speed with which individuals reduce their periodic oscillation around their equilibrium point); $\dot{x}_t =$ the rate at which religiousness is changing; $\eta =$ the square of the frequency of oscillation; $x_t =$ religiousness at time t; and $e_t =$ error in measuring the second derivative of religiousness at time t.

The only difference between a one-attractor model and a two-attractor model is the inclusion of a cubic term as a predictor of the second derivatives:

$$\ddot{\mathbf{x}} = \eta x_t + \zeta \dot{\mathbf{x}}_t + \xi x_t^3 + e_t \tag{2}$$

Including the x^3 term allows religiousness to be attracted toward two equilibria instead of one.

It is possible to test the viability of these two models using the repeated measures of religious saliency that we have developed for participants in the Terman Life Cycle Study of Children With High Ability (Terman & Oden, 1947).

Participants

Over the last few years, the first author and colleagues have been using data from the Terman Life Cycle Study of Children With High Ability to shed light on questions related to the development of religiousness over the life

course (McCullough et al., 2005; McCullough, Tsang, & Brion, 2003). In this previous work, we have used longitudinal research designs that are based on the assumption that an individual's religiousness at a given point in time is dependent on his or her age rather than upon a dynamical notion of how people may self-regulate religiousness to maximize person-environment fit.

The Terman study comprises data from 1,528 bright and gifted boys and girls (all of the students had intelligence quotients exceeding 135) from the state of California. The average birth year for children in the original sample was 1910. Since the study was initiated, participants have been recontacted for more than a dozen follow-up surveys.

For the present study, we used 957 (approx. 56% male, 44% female; ages in 1940 ranged from 24 to 40 yrs) of the 1,528 original participants. As of 1940, these mostly white, middle-class adults were highly educated (approximately 99% had high school diplomas; 89% had at least some college experience, 70% had at least a bachelor's degree; 45% had at least a master's degree, and 8% had a doctorate or more). Most (approximately 65%) were married (approximately 31% were single and 3% were divorced).

Measures of Religiousness

Although Terman and successive directors of the Terman longitudinal study collected a great deal of data on participants' religious lives, including dozens of items in checklist or Likerttype format, none of these items was repeated in exactly the same way across surveys. Such frustrations are not uncommon in longitudinal work (Elder, Pavalko, & Clipp, 1993), but social scientists have found a productive way to cope with them.

As in other recent work on religious development (Wink & Dillon, 2001, 2002) we used a recasting method (Elder et al., 1993) to develop a five-point rating scale for measuring the saliency or importance of religion to participants (which we called religious saliency). This measure is conceptually similar to other measures of religious saliency that have been used in previous longitudinal research on religious development among adults (e.g., Argue et al., 1999; Wink & Dillon, 2001). To use these rating scales, trained raters read all information that participants provided regarding their religiousness for surveys that Terman and associates conducted in 1940, 1950, 1960, 1977, 1986, and 1991. After reading the religious information on a given participant for a given year, raters then provided a single numeric rating of their perceptions of the participant's religious saliency at that point in the participant's life. Scores on this scale ranged from -1 = participarticipant is actively antireligious, noted by lack of personal religious interest/inclination, total lack of life satisfaction gained from religion, and some degree of hostility/suspicion toward religion or religious beliefs, to 4 = religion has very high importance in participant's life, as noted by very high interest in religion, very high religious inclination, or very high degree of life satisfaction gained from religion. Interrater reliability was very good (McCullough et al., 2005).

Results

Does a Visual Display of the Data Suggest That Dynamical Processes May Be at Work?

A good way to begin a project designed to examine self-regulatory processes is to acquire an inductive sense of how change actually occurs in the data. Visual displays of data are important for this purpose. Figure 26.5 is an empirical slope field of our religious saliency data (Boker & McArdle, 2005). Based on an aggregation of data from the entire sample, this figure portrays the expected slopes (that is, the rates of change) in religious saliency for combinations of age and the measured values of the variable. That is, this plot depicts the direction in which, and the rate at which, scores on the variable are likely to change after a small amount of time has elapsed. In a sense, they depict the flow of scores with the passage of time.

The empirical slope field in figure 26.5 shows that for most people, religious saliency has a fairly steep positive slope from about age 20 to about age 50, with the slopes appearing slightly steeper for individuals who score in the range -1 < x < 0 on religious saliency around age 20. As time passes, however, the slopes become increasingly flat for people at low levels of religious saliency (i.e., people with religious saliency scores around x = 1), and increasingly negative for individuals with moderate levels



Figure 26.5 Empirical slope field for religious saliency.

of religious saliency (i.e., people scoring in the range 1 < x < 3). This suggests that religious saliency may possess a single attractor somewhere in the lower range of the scale (i.e., people scoring in the range -.5 < x < 1). There is one region of the slope field plot for which low levels of religious saliency do not appear to operate as a strong attractor: the upper right corner. This region describes individuals who possess very high religious saliency scores into their mid-50s and beyond. For individuals in this region, changes in religious saliency with small changes in time are essentially zero, suggesting that religious saliency has stopped changing and is maintaining a consistently high level through the remainder of the life course. This raises the possibility that there may actually be two attractors in this system: an attractor for religious saliency scores in the range (.5 <x < 1) and a second attractor around x = 4.0.

Testing Second-Order Differential Equations

With a feel for the data that led us to suspect that the interindividual variations in religious saliency might be created by a self-regulatory system in which people's scores were oscillating (with damping) toward one or possibly two attractors, we proceeded to test some formal differential equations. Our first model was a second-order differential equation in the form of Equation 1.

Calculating First and Second Derivatives With Local Linear Approximation

To generate the necessary first and second derivatives, we used local linear approximation (LLA; Boker & Nesselroade, 2002), which involves estimating the first derivatives (i.e., slopes or rates of linear change) and second derivatives (e.g., rates of acceleration or curvature) for an observed value of interest based on the observed data preceding and following the observed value of interest. Therefore, estimating these derivatives requires at least three occasions of measurement per individual. Since we measured religious saliency on up to six occasions per individual, we could estimate up to four of the necessary triads (i.e., an observed value and its corresponding first and second derivatives) if we based our estimates on data drawn $\tau = 1$ time step (or measurement occasion) before and after the value for which we wish to approximate the first derivative and second derivative.

However, it is often advantageous to also develop measures of the first and second derivatives using a $\tau > 1$ (i.e., with data more than one time step or occasion of measurement before and after the measured value of interest). Using a value of $\tau > 1$ tends to help reduce the influence of measurement error by low-pass filtering the data while estimating the derivatives. Measurement error will show up as if it were an oscillation with a period of two occasions of measurement. Using observations {*x*1, *x*3, *x*5} to calculate one set of derivatives and {*x*2, *x*4, *x*6} to calculate a second set of derivatives means that the measurement error will tend to cancel itself out in the long run.

In such cases, to estimate the first and second derivatives for any measured value, we will look not one occasion to the left and right of the index value, but two or more occasions to the left and the right of the value of interest. So, for the analyses that we conducted, we estimated first and second derivatives using values of $\tau = 1$ (these models are referred to below as Models 1a, 2a, 3a, and 4a) and compared those results to the results that emerged from using values of $\tau = 2$ (these models are referred to below as Models 1b, 2b, 3b, and 4b).

We actually calculated four sets of models. The first and second models were second-order models that posited the existence of one point attractor. These two models differed only in the fact that in Model 1, we posited that the parameter estimates of η and ζ were the same for all individuals; that is, that there existed an underlying dynamical system that generalized to all subjects (Boker & Nesselroade, 2002). In contrast, in Model 2 we posited that these parameters varied across persons.

Models 3 and 4 were two-attractor models that posited two point attractors rather than a single one. Model 3 posited that the parameter estimates of η and ζ were the same for everyone, whereas they were permitted to differ across individuals in Model 4.

Models 1a and 1b: A Second-Order Model With One Attractor and Fixed Effects

In testing our first model—a second-order differential equation with one attractor—we assumed that every individual had the same parameters for η (the frequency parameter) and ζ (the damping parameter). Simultaneously regressing \ddot{x}_t upon its corresponding values for x and \dot{x}_t as in Equation 1 yields the parameters that appear in the Model 1a and 1b columns of table 26.1. The fact that the coefficient for ζ is negative for Model 1a indicates that some damping appears to be occurring in this self-regulatory system: As time passes, the amount of oscillation around the mean becomes smaller-much like a pendulum with friction. In other words, religious saliency looks like a resilient system that manifests less and less fluctuation as time passes. Had the coefficient for ζ been positive, this would have suggested that religiousness is an excitable system in which fluctuation actually becomes greater and greater with the passage of time. The value of η was negative near zero, indicating that the system does not oscillate rapidlyin line with our expectations given the slow change observed in the empirical slope field in figure 26.5.

However, Model 1a appears inadequate, accounting for only 14% of the variance in the second derivative. This is in line with what one would expect when a slow-frequency system is evaluated with a value of $\tau = 1$ (Boker & Nesselroade, 2002). There is a built-in dependence between the value of *x* and \ddot{x}_t that results from the LLA method of calculating derivatives. This dependence results in an expected $R^2 = 0.65$ when a second-order linear differential equation model (as we use here) is applied to normally distributed random numbers (Boker & Nesselroade, 2002). In essence, this surprising result is due to the fact that normally distributed numbers will appear to oscillate with a period

TABLE 26.1 Parameter Estimates for Differential Equation Models of Religious Saliency

	Model							
	1a	1b	2a	2b	3a	3b	4a	4b
	One	One	One	One	Two	Two	Two	Two
	Attractor,							
	Fixed	Fixed	Random	Random	Fixed	Fixed	Random	Random
	Effects							
	$(\tau = 1)$	$(\tau = 2)$						
ζ	-0.655	0.029	-0.614	-0.063	-0.654	-0.039	-0.620	-0.021
	-0.214	-0.113	-0.206	-0.117	-0.071	-0.109	-0.060	-0.103
η ξ R^2					-0.012	-0.001	-0.013	-0.001
	0.14	0.34	0.30	0.72	0.147	0.612	0.310	0.720
$\eta \tau^2$	-0.21	-0.45	-0.21	-0.47	-0.07	-0.44	-0.06	-0.41
Number of								
observations	2,417	869	2,417	869	2,417	869	2,417	869
Number of								
individuals	957	571	957	571	957	571	957	571

equal to twice the value of t used to estimate the model. However, the dependence between t and frequency results in an expected value of $\eta\tau^2 = -2.0$. We can use this fact to calculate values of $\eta\tau^2$ for Models 1a and 1b of -0.21and -0.45, respectively. These values are far from -2.0 and indicate that the estimated period of oscillation is much slower than the expected value for normally distributed random numbers. Nevertheless, even Model 1b, which evaluated the one-attractor fixed effects model with $\tau = 2$, was not very good, accounting for only 34% of the variance in the second derivative.

Models 2a and 2b: A Second-Order Model With a Single Attractor and Random Effects

Next we evaluated a second-order differential equation with one attractor, as in Models 1a and 1b, but we also allowed the η (frequency) and ζ (damping) parameters to vary across persons. In other words, in these models we evaluated the possibility that the differences in the trajectories observed for each individual were produced not only by individual differences in initial conditions (initial displacement from equilibrium and initial rate of change), but also individual differences in frequency and damping.

Using $\tau = 1$ (Model 2a), we saw a small increase in the variance accounted for, $R^2 = .30$, but a much more impressive increase in model fit occurred when we conducted the same model using $\tau = 2$ (Model 2b). In this version of the oneattractor model with random effects, the model accounted for 72% of the variation in the second derivative of religious saliency. This sizeable increase in R^2 not only suggests that the secondorder model with one attractor and random effects may provide an acceptable fit to the data, but it also suggests that using $\tau = 2$ for calculating the first and second derivatives introduces less bias to the estimates than does calculating them with $\tau = 1$ (Boker & Nesselroade, 2002). Since the value for $\eta \tau^2$ was well below 2, we need not be concerned that the fluctuations were solely the product of noise.

The length of time, λ , that elapses during one full cycle of religious saliency (also called the period) can be calculated from the estimated η parameter

$$\lambda = 2\pi \sqrt{1/-\eta}, \qquad (3)$$

where 2π converts from units of radians into units of time. Using the average values $\eta =$ -.206 and -.117 estimated from Models 2a and 2b, and recalling that time was expressed in decades, we estimate that the average period for a full cycle of religious saliency is between 140 and 180 years.

Models 3a and 3b: A Second-Order Model With Two Attractors and Fixed Effects

Because our visual inspection of the slope field plot for religious saliency hinted that a second attractor might be present, we ran a third model that permitted individuals' scores to be attracted to either of two attractors. This was accomplished by adding a coefficient to the models representing x^3 . The results of these models appear as Model 3a and Model 3b in table 26.1. Using R^2 as a measure of goodness of fit, these models did not fit appreciably better than did the oneattractor models with random effects (Models 2a and 2b).

Model 4: A Second-Order Model With Two Attractors and Random Effects

In this version of the two-attractor model model, we allowed the ζ and x^3 parameters (representing the relative strength of the two attractors) to vary between persons. The coefficients for η were not random because their high correlation with x^3 (r = -.92). The coefficients for these models appear as Model 4a and Model 4b in table 26.1. Using R^2 as a measure of goodness of fit, these models did not fit better than did the one-attractor models with random effects (Models 2a and 2b).

Discussion

In writing this chapter, we wished to introduce readers to multivariate methods for studying selfregulation using differential equation models. We used a real data set that allowed us to examine the self-regulation of religiousness across the adult life course. As chapters on religion and spirituality in some of positive psychology's seminal volumes (Mattis, 2004; Pargament & Mahoney, 2002; Tsang & McCullough, 2003) testify, religion and spirituality are constructs with considerable relevance to the burgeoning field of positive psychology.

Using recently developed methods for studying self-regulatory systems with multiwave panel data (Boker & Ghisletta, 2001; Boker & Nesselroade, 2002), we tested several models that allowed us to examine the possibility that the importance an individual ascribes to religion is to some extent governed by the functioning of an internal guidance system that seeks to move people toward an equilibrium value. The results of the best-fitting of these models-a singleattractor model with random effects for the damping and frequency parameters-suggest that it is indeed plausible to posit that religiousness is, to some degree, self-regulatory in nature. Over the course of adulthood, individuals appear to be adjusting their levels of religiousness toward equilibrium values. As people approach their points of equilibrium, the oscillation in their religious trajectories becomes less pronounced. According to our analyses, an optimal level of religiousness, that is, one that provided equilibrium, was somewhere around a value of 1 on a scale ranging from 0 to 4. A value of 1 represents a fairly low level of religiousness, which is not terribly surprising given the fact that the individuals in this sample were considerably less religious, on average, than were the general population at large (McCullough et al., 2003). However, the indication of individual differences in coefficients for damping and frequency suggest that this single equilibrium value may be a misleading portrayal of particular individuals' equilibrium values (Boker & Nesselroade, 2002).

We were somewhat surprised to find that a two-attractor model did not perform any better than the one-attractor model, as our visual inspection of the slope field plot suggested the possibility of a second point attractor around religiousness values of 4. However, the analyses did not give any reason to favor the less parsimonious two-attractor model over the oneattractor model. Studies with greater numbers of observations per individual would have helped us to gain greater statistical power and, thus, perhaps a greater chance of detecting a second attractor if one truly existed.

As mentioned above, the fact that a model incorporating random effects provided a better fit to the data than did a model with fixed effects means that individuals differed in their damping rates and frequencies of oscillation. Individual differences in damping reflect differences in the extent to which individuals' religious systems can impose friction upon the intrinsic oscillation that the system is also producing, thereby reducing the amount of swing above and below the equilibrium point with each oscillation. Individual differences in frequency represent individual differences in the intrinsic cycling rate of individuals' religious self-regulatory systems; that is, the number of oscillations completed in a given amount of time. Finally, individual differences in initial conditions represent individual differences between persons such that some people began an observation period with higher levels of religiousness (i.e., high levels of initial displacement from their equilibrium values) than did other people, or more positive slopes (i.e., steep upward initial trajectories) than did other people. In the self-regulation framework we have described herein, it is these individual parametric differences, along with individual differences in initial conditions, that explain the variety of longitudinal trajectories that are seen among the individuals in the Terman study.

Possible Next Steps

Having found individual differences in frequency and damping, as well as individual differences in initial conditions, it might be worthwhile to attempt to account for these individual differences. What factors might cause some individuals to experience more or less damping, or faster or slower frequencies of oscillation, than do others? Can these individual differences be attributed to individual differences in personality? Perhaps individuals with greater emotional stability experience less dramatic fluctuation around their equilibria than do others. Alternatively, perhaps people who marry spouses with levels of religiousness that are similar to their own experience greater damping-that is, greater efficiency in reducing the amount of swing around equilibrium values. To explain individual differences in initial conditions, we might look to background factors such as the degree to which individuals' parents themselves were religiously devout, which might have produced individual differences in initial conditions. Or perhaps we could look to their religious histories in adolescence to find evidence that they had undergone conversion experiences that produced positive religious slopes in early adulthood.

Differential equation modeling of selfregulatory processes is appealing in part because of the elegant way in which it produces estimates of psychologically meaningful processes. Positing that people have different damping and

frequency parameters rests on the strong assumption that damping mechanisms and intrinsic frequencies actually exist somewhere under the human skin. In comparison, consider the latent growth parameters that might be used to depict the same longitudinal data in a multilevel growth curve model (Raudenbush & Bryk, 2002). Hypothesizing that interindividual differences in religious development are produced by interindividual differences in initial status values, rates of linear change, or degrees of curvature over a bounded interval, as one might in a growth curve model, does not yield parameters that have intrinsic psychological meaning: One is still left asking what mechanisms produced the interindividual differences in initial status, or linear change, or curvature. This is not to say that growth curve models are not important tools or that differential equation models are a cure-all for modeling longitudinal data. Each type of model has its place and they address rather different questions. Nevertheless, differential equation models have considerable potential to shed light on how people change.

Limitations of the Data Set and Design Recommendations for Researchers

In some respects, the Terman data set was less than ideal for testing hypotheses about selfregulation. With a maximum of six observations per person (data on people's religiousness were available from 1940, 1950, 1960, 1977, 1986, and 1991), it was possible to build a maximum of four observations per person for which a displacement, first derivative, and second derivative could be calculated (since each observation's first and second derivatives could only be calculated if values existed before and after the observation in question). Using $\tau = 2$ —the degree of spacing between observations that provided us with the best fit to the data-a maximum of two observations per person were available for which the necessary triads could be established for estimating differential equations. This cut our number of observations by two thirds and the number of participants by 40%. By increasing the number of observations per person, statistical power for conducting these models would have increased, as would the analytic options and the range of dynamical questions we could have asked.

Second, because we were restricted to singleitem measures of religiousness, it was impossible to control the effects of measurement error, which might have been considerable. With two or more indicators of the construct at each time point, we could have reduced measurement error by working with aggregates of observed variables. As a result, our models would have demonstrated greater power to account for individual variation in the \ddot{x}_t values.

Third, we could have developed a better understanding of religious development from these data if we were working in reference to a known perturbation in people's religious self-regulatory systems. The "pendulum with friction" is a common conceit used to frame inquiries into the self-regulatory dynamics of systems. It is easier to understand the behavior of a pendulum if we know when—and from what height—the pendulum was released. That is, it is useful to know when the perturbation occurred and how large it was.

Extending the pendulum conceit to the domain of religious change, it might be easier to understand the self-regulatory mechanisms underlying religious development if one worked with data collected before and after a known disruption to individuals' religious lives. For instance, one might study a group of individuals who had recently experienced a religious conversion as the result of attending a religious event. Alternatively, one might study religious or spiritual responses to tragedy. Recent evidence suggests that adults in the United States became, on average, slightly more spiritually inclined in the months following the terrorist attacks of September 11, 2001 (Peterson & Seligman, 2003), and there is good experimental evidence that exposing people to tragedies in the laboratory increases a questing, open-ended approach to religion (Burris, Jackson, Tarpley, & Smith, 1996; Krauss & Flaherty, 2001). The death of a spouse also appears to create temporary perturbations in widows' and widowers' religious functioning (Brown, Nesse, House, & Utz, 2004). Presumably, insofar as self-regulation actually occurs in the religious domain, these perturbations from equilibrium triggered the operation of that self-regulatory system. Laboratory methods such as those developed by Burris et al. (1996) might be used to introduce systematic religious perturbations, and differential equation models might then be used to estimate mechanisms by which people modulate the effect of the perturbations, thereby allowing researchers to explicitly test self-regulatory hypotheses about religious change.

Summary

Self-regulation is a useful concept for a comprehensive positive psychology. Goals, forgiveness, resilience, posttraumatic growth, and hardiness are but a few of the concepts central to positive psychology that lend themselves to a self-regulatory conceptualization. We hope that the present chapter has provided a brief introduction to the promise that these models might hold for theoretical work and new empirical studies in this young and promising field.

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References

- Argue, A., Johnson, D. R., & White, L. K. (1999). Age and religiosity: Evidence from a three-wave panel analysis. *Journal for the Scientific Study* of *Religion*, 38, 423–435.
- Bahr, H. M. (1970). Aging and religious disaffiliation. Social Forces, 49, 59-71.
- Bisconti, T. L. (2001). Widowhood in later life: A dynamical systems approach to emotion regulation. Unpublished doctoral dissertation, University of Notre Dame.
- Boker, S. M. (2001). Differential structural equation modeling of intraindividual variability. In L. M. Collins & A. G. Sayer (Eds.), New methods for the analysis of change (pp. 5–27).

Washington, DC: American Psychological Association.

- Boker, S. M., & Bisconti, T. L. (2006). Dynamical systems modeling in aging research. In C. S. Bergeman & S. M. Boker (Eds.), Quantitative methodology in aging research (pp. 185–229). Mahwah, NJ: Erlbaum.
- Boker, S. M., & Ghisletta, P. (2001). Random coefficients models for control parameters in dynamical systems. *Multilevel Modelling Newsletter*, 13(1), 10–17.
- Boker, S. M., & Graham, J. (1998). A dynamical systems analysis of substance abuse. *Multivariate Behavioral Research*, 33, 479–507.
- Boker, S. M., & McArdle, J. J. (2005). Vector field plots. In P. Armitage & T. Colton (Eds.), Encyclopedia of Biostatistics: Vol. 8 (2nd ed., pp. 5700–5704). Chichester, UK: Wiley.
- Boker, S. M., & Nesselroade, J. R. (2002). A method for modeling the intrinsic dynamics of intraindividual variability: Recovering the parameters of simulated oscillators in multi-wave panel data. *Multivariate Behavioral Research*, 37, 127–160.
- Brown, S. L., Nesse, R. M., House, J., & Utz, R. L. (2004). Religion and emotional compensation: Results from a prospective study of widowhood. *Personality and Social Psychology Bulletin*, 30, 1165–1174.
- Burris, C. T., Jackson, L. M., Tarpley, W. R., & Smith, G. J. (1996). Religion as quest: The selfdirected pursuit of meaning. *Personality and Social Psychology Bulletin*, 22, 1068–1076.
- Carver, C. S., & Scheier, M. F. (1998). On the selfregulation of behavior. Cambridge, UK: Cambridge University Press.
- D'Onofrio, B. M., Eaves, L. J., Murrelle, L., Maes, H. H., & Spilka, B. (1999). Understanding biological and social influences on religious affiliation, attitudes, and behaviors: A behavior genetic perspective. *Journal of Personality*, 67, 953–984.
- Elder, G. H., Jr., Pavalko, E. K., & Clipp, E. C. (1993). Working with archival data: Studying lives. Newbury Park, CA: Sage.
- Flor, D. L., & Knapp, N. F. (2001). Transmission and transaction: Predicting adolescents' internalization of parental religious values. *Journal* of Family Psychology, 15, 627–645.
- Fowler, J. (1981). Stages of faith: The psychology of human development and the quest for meaning. New York: Harper and Row.
- Hall, G. S. (1904). Adolescence, its psychology and its relations to physiology, anthropology, sociology, sex, crime, religion and education (Vol. 1). New York: D. Appleton.

- Higgins, E. T., Grant, H., & Shah, J. (1999). Self-regulation and quality of life: Emotional and non-emotional life experiences. In D. Kahneman, E. Diener, & N. Schwartz (Eds.), Well-being: The foundations of hedonic psychology (pp. 244–266). New York: Russell Sage.
- Ingersoll-Dayton, B., Krause, N., & Morgan, D. (2002). Religious trajectories and transitions over the life course. *International Journal of Aging and Human Development*, 55, 51–70.
- Kelley-Moore, J. A., & Ferraro, K. F. (2001). Functional limitations and religious service attendance in later life: Barrier and/or benefit mechanism? *Journal of Gerontology: Social Sciences*, 56B, S365–S373.
- Krauss, S. W., & Flaherty, R. W. (2001). The effects of tragedies and contradictions on religion as a quest. *Journal for the Scientific Study of Religion*, 40, 113–122.
- Masten, A. S., & Reed, M. J. (2002). Resilience in development. In S. J. Lopez (Ed.), *Handbook of positive psychology* (pp. 74–88). New York: Oxford University Press.
- Mattis, J. (2004). Spirituality. In C. Peterson & M.
 E. P. Seligman (Eds.), *Character strengths and virtues: A handbook and classification* (pp. 599–622). New York: Oxford University Press/American Psychological Association.
- McArdle, J. J., & Hamagami, F. (2003). Structural equation models for evaluating dynamic concepts within longitudinal twin analyses. *Behavior Genetics*, 33, 137–159.
- McCullough, M. E., Enders, C. K., Brion, S. L., & Jain, A. R. (2005). The varieties of religious development in adulthood: A longitudinal investigation of religion and rational choice. *Journal of Personality and Social Psychology*, 89, 78–89.
- McCullough, M. E., Tsang, J., & Brion, S. (2003). Personality traits in adolescence as predictors of religiousness on early adulthood: Findings from the Terman Longitudinal Study. *Personality* and Social Psychology Bulletin, 29, 980–991.
- McCullough, M. E., & Witvliet, C. v. O. (2002). The psychology of forgiveness. In S. J. Lopez (Ed.), Handbook of positive psychology (pp. 446–458). New York: Oxford University Press.

- Oud, J. H. L., & Jansen, R. A. R. G. (2000). Continuous time state space modeling of panel data by means of SEM. *Psychometrica*, 65, 199–215.
- Pargament, K. I., & Mahoney, A. (2002). Spirituality: Discovering and conserving the sacred. In C. R. Snyder & S. J. Lopez (Eds.), *Handbook of positive psychology* (pp. 646–659). New York: Oxford University Press.
- Peterson, C., & Seligman, M. E. P. (2003). Character strengths before and after September 11. *Psychological Science*, 14, 381–384.
- Raudenbush, S. W., & Bryk, A. S. (2002). Hierarchical linear models: Applications and data analysis methods (2nd ed.). Thousand Oaks, CA: Sage.
- Sasaki, M., & Suzuki, T. (1987). Changes in religious commitment in the United States, Holland and Japan. *American Journal of Sociology*, 92, 1055–1076.
- Selye, H. (1956). *The stress of life*. New York: McGraw-Hill.
- Sherkat, D. E. (1998). Counterculture or continuity: Competing influences on baby boomers' religious orientations and participation. *Social Forces*, 76, 1087–1115.
- Stolzenberg, R. M., Blair-Loy, M., & Waite, L. J. (1995). Religious participation in early adulthood: Age and family life cycle effects on church membership. *American Sociological Review*, 60, 84–103.
- Terman, L. M., & Oden, M. H. (1947). Genetic studies of genius IV: The gifted child grows up. Stanford, CA: Stanford University Press.
- Tsang, J., & McCullough, M. E. (2003). Measuring religious constructs: A hierarchical approach to construct organization and scale selection. In C. R. Snyder (Ed.), *Handbook of positive psychological assessment* (pp. 345–360). Washington, DC: American Psychological Association.
- Wink, P., & Dillon, M. (2001). Religious involvement and health outcomes in late adulthood. In T. G. Plante & A. C. Sherman (Eds.), *Faith and health* (pp. 75–106). New York: Guilford.Wink, P., & Dillon, M. (2002). Spiritual development across the adult life course: Findings from a longitudinal study. *Journal of Adult Development*, 9, 79–94.